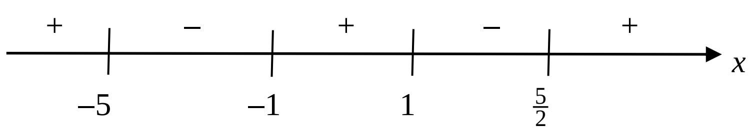
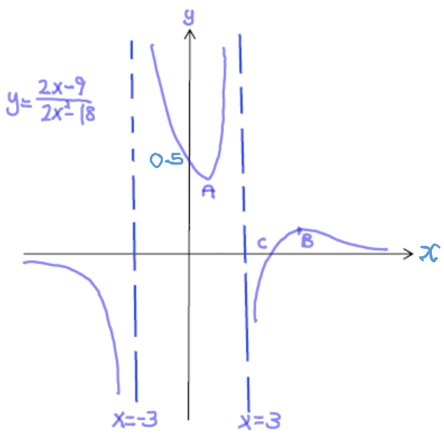
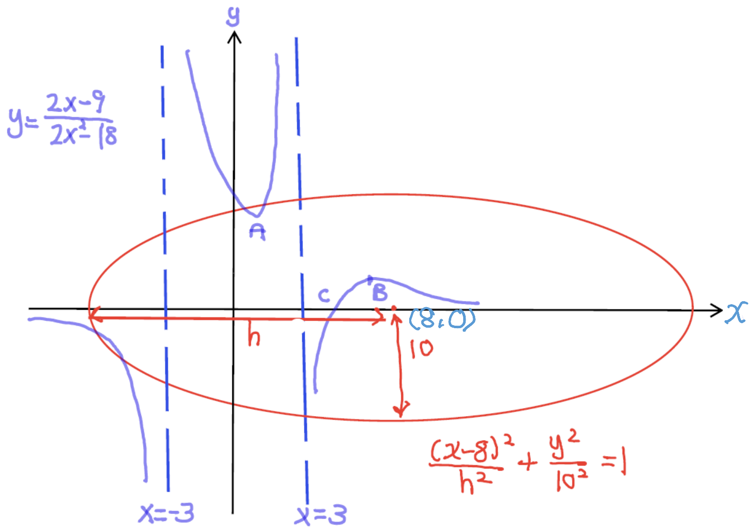
2019 RVHS H2 Maths Prelim P1 Solutions

1	Solution [5] Inequality	
	$\frac{2x}{x+5} < \frac{1}{x-1} \Rightarrow \frac{2x}{x+5} - \frac{1}{x-1} < 0$ --------------- (1) Then, we have $\frac{2x^2 - 2x - x - 5}{(x+5)(x-1)} < 0$ $\frac{2x^2 - 3x - 5}{(x+5)(x-1)} < 0$ $\frac{(2x-5)(x+1)}{(x+5)(x-1)} < 0$ Applying number line test:  Therefore, the solution to inequality (1) is $-5 < x < -1 \quad \text{or} \quad 1 < x < \frac{5}{2}$.	
	Next, we replace ‘ x ’ by ‘ $\lvert x \rvert$ ’ in inequality (1) to obtain $\frac{2\lvert x \rvert}{\lvert x \rvert + 5} - \frac{1}{\lvert x \rvert - 1} < 0$ -------------- (2) Thus, the solution to inequality (2) correspondingly is $-5 < \lvert x \rvert < -1 \quad \text{or} \quad 1 < \lvert x \rvert < \frac{5}{2}$ i.e. $-5 < \lvert x \rvert < -1$ (NA) or $1 < \lvert x \rvert < \frac{5}{2}$ Thus, the solution to inequality (2) is $-\frac{5}{2} < x < -1 \quad \text{or} \quad 1 < x < \frac{5}{2}$.	

2	Solution [8] Maclaurin's Series	
	(i) $\frac{dy}{dx}=\frac{\cos x}{2y}$ $2y\frac{dy}{dx}=\cos x$ Diff Implicitly w.r.t x, $2y\frac{d^2y}{dx^2}+2\frac{dy}{dx}\frac{dy}{dx}=-\sin x$ $2y\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^2=-\sin x$ When x = 0, y = 2 (GIVEN) Hence $\frac{dy}{dx}=\frac{1}{4}$ $(2)(2)\frac{d^2y}{dx^2}+2\left(\frac{1}{4}\right)^2=0$ $\frac{d^2y}{dx^2}=-\frac{1}{32}$ Using the Maclaurin's formula, $y=2+x(\frac{1}{4})+\frac{x^2}{2!}(-\frac{1}{32})+...$ $\approx 2+\frac{x}{4}-\frac{x^2}{64}$(up to the x^2term) Equation of tangent at $x=0$: $y=2+\frac{1}{4}x$	
	(ii) $\frac{dy}{dx}=\frac{\cos x}{2y}$ $\int 2y\ dy=\int \cos x\ dx$ $y^2=\sin x+C$ Subst $(0,2)$, then $C=4$ $y=\pm\sqrt{4+\sin x}$ Since $x=0$ and $y=2$, $\therefore y=\sqrt{4+\sin x}$	

	(iii) $y=\sqrt{4+\sin x}$ $=(4+\sin x)^{\frac{1}{2}}$ $\approx(4+x)^{\frac{1}{2}}$ (since x is small) $=2\left(1+\frac{x}{4}\right)^{\frac{1}{2}}$ $=2\left(1+\frac{1}{2}.\frac{x}{4}+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(\frac{x}{4}\right)^2}{2!}+...\right)$ $\approx 2+\frac{x}{4}-\frac{x^2}{64}$	

3	Solution [8] Curve Sketching	
	(i) Since $x = 3$ is an asymptote, $2(3)^2 - b = 0$ $b = 18$ C passes through $\left(\frac{9}{2}, 0\right)$ implies $2\left(\frac{9}{2}\right) - a = 0$ $a = 9$	
	(ii)  A(1.15, 0.436) B(7.85, 0.0637) C(4.50, 0.00) TO find stationary points, set $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ And work through the maths to do it. There should be 2 stationary points. This is actually the most important part. The rest is the axes intercepts (let x=0 and then let y=0), identify the asymptotes. And the shape is important – especially the part on moving as close to the asymptote as possible.	

	(iii)  $\frac{(x-8)^2}{h^2}+\frac{y^2}{10^2}=1$ is the ellipse with centre at $(8,0)$, with axes of length h and 10. Horizontal width need to be at least $8+4=12$ units. Therefore $h\geq 12$, for 6 distinct points of intersections.	

4	Solution [10] Functions	
	(i) Let $y=\dfrac{ax+b}{cx-a}, x\in\mathbb{R}, x\neq\dfrac{a}{c}$ $y(cx-a)=ax+b$ $x(cy-a)=ay+b$ $x=\dfrac{ay+b}{cy-a}$ Replacing y by x, $\therefore \mathrm{f}^{-1}(x)=\dfrac{ax+b}{cx-a}, x\in\mathbb{R}, x\neq\dfrac{a}{c}$ Since $\mathrm{f}(x)=\mathrm{f}^{-1}(x) = \dfrac{ax+b}{cx-a}$, f is self-inverse. (SHOWN)	
	(ii) $\mathrm{f}(x)=\mathrm{f}^{-1}(x)$ Composing function f on both sides, $\mathrm{ff}(x)=\mathrm{ff}^{-1}(x)$ $\mathrm{f}^2(x)=x$ $\mathrm{D}_{\mathrm{f}^2}=\mathbb{R}\setminus\left\{\dfrac{a}{c}\right\}, \quad \mathrm{R}_{\mathrm{f}^2}=\mathbb{R}\setminus\left\{\dfrac{a}{c}\right\}$ or present as $\mathrm{R}_{\mathrm{f}^2}=\left(-\infty,\dfrac{a}{c}\right)\cup\left(\dfrac{a}{c},\infty\right)$	
	(iii) $\mathrm{f}^{-1}(x)=x$ $\dfrac{ax+b}{cx-a}=x$ $ax+b=cx^2-ax$ $cx^2-2ax-b=0$ $x^2-\dfrac{2a}{c}x-\dfrac{b}{c}=0$ $\left(x-\dfrac{a}{c}\right)^2=\dfrac{b}{c}+\left(\dfrac{a}{c}\right)^2$	

	$x-\frac{a}{c}=\pm\sqrt{\frac{bc+a^2}{c^2}}$ $x=\frac{a}{c}+\sqrt{\frac{bc+a^2}{c^2}}$ or $\frac{a}{c}-\sqrt{\frac{bc+a^2}{c^2}}$ $=\frac{a+\sqrt{bc+a^2}}{c}$ or $\frac{a-\sqrt{bc+a^2}}{c}$	
	(iv) Now, a =2, b = 5 and c = 3 $\mathrm{f}(x)=\frac{2x+5}{3x-2}, x\in\mathbb{R}, x\neq\frac{2}{3}$ $\mathrm{g}(x)=\mathrm{e}^x+2, x\in\mathbb{R}$ FACT: For fg to exist , need $\mathrm{R_g}\subseteq \mathrm{D_f}$ to hold , $\mathrm{R_g}=(2,\infty)\subseteq \mathbb{R}\setminus\left\{\frac{2}{3}\right\}=\mathrm{D_f}$ fg does exist.	
	(v) $D_g=\mathbb{R}\overset{g}{\mapsto}R_g=(2,\infty)\overset{\mathrm{f}}{\mapsto}R_{\mathrm{fg}}=\left(\frac{2}{3},\frac{9}{4}\right)$ Therefore Range of fg is $\left(\frac{2}{3},\frac{9}{4}\right)$	

5	Solution [7] MOD	
	(i) $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ $= \frac{e^x(1-e^{-2x})}{e^x(1+e^{-2x})}$ $= \frac{1-e^{-2x}}{1+e^{-2x}}$	
	(ii) $f(n+1) - f(n) = [\sinh x][\text{sech}\left(n+\frac{1}{2}\right)x][\text{sech}\left(n-\frac{1}{2}\right)x]$ $\sum_{n=1}^{N} f(n+1) - f(n) = (\sinh x)\sum_{n=1}^{N}[\text{sech}\left(n+\frac{1}{2}\right)x][\text{sech}\left(n-\frac{1}{2}\right)x]$ $\sum_{n=1}^{N}[\text{sech}\left(n+\frac{1}{2}\right)x][\text{sech}\left(n-\frac{1}{2}\right)x] = \frac{1}{\sinh x}\sum_{n=1}^{N} f(n+1) - f(n)$ $S_n = \frac{1}{\sinh x}\sum_{n=1}^{N} f(n+1) - f(n)$ $= \frac{1}{\sinh x}\begin{bmatrix} \cancel{f(2)} & - & f(1) \\ \cancel{f(3)} & - & \cancel{f(2)} \\ \cancel{f(4)} & - & \cancel{f(3)} \\ & \ldots & \\ \cancel{f(N)} & - & \cancel{f(N-1)} \\ f(N+1) & - & \cancel{f(N)} \end{bmatrix}$ $= \frac{1}{\sinh x}\left(f(N+1) - f(1)\right)$ $= (\text{cosech } x)\left(\tanh(N+\frac{1}{2})x - \tanh(\frac{1}{2})x\right)$ $\therefore A = N + \frac{1}{2}$	
	(iii) S_∞ $= \lim_{N\to\infty}\sum_{n=1}^{N}\left[\text{sech}\left(n+\frac{1}{2}x\right)\right]\left[\text{sech}\left(n-\frac{1}{2}x\right)\right]$ $= \lim_{N\to\infty}(\text{cosech } x)\left[\tanh\left(N+\frac{1}{2}\right)x - \tanh\left(\frac{1}{2}\right)x\right]$	

	Consider $\lim_{N\to\infty} \tanh\left(N+\frac{1}{2}\right)x$ $= \lim_{N\to\infty} \frac{1-e^{-2\left(N+\frac{1}{2}\right)x}}{1+e^{-2\left(N+\frac{1}{2}\right)x}}$ $=1$ Since $\tanh\left(N+\frac{1}{2}\right)x \to 1$ as $N\to\infty$, therefore S_∞ exists. S_∞ $=\left(\operatorname{cosech} x\right)\left[1-\tanh\left(\frac{1}{2}\right)x\right]$ $P=1,\ Q=\frac{1}{2}$	

6	Solution [11] Complex Numbers	
	(a) $(1+\mathrm{i})z^2 - z + (2-2\mathrm{i}) = 0$ $z = \dfrac{1 \pm \sqrt{1-4(1+\mathrm{i})(2-2\mathrm{i})}}{2+2\mathrm{i}}$ $= \dfrac{1 \pm \sqrt{1-8(1+\mathrm{i})(1-\mathrm{i})}}{2+2\mathrm{i}}$ $= \dfrac{1 \pm \sqrt{1-8(2)}}{2+2\mathrm{i}}$ $= \dfrac{1 \pm \sqrt{15}\mathrm{i}}{2+2\mathrm{i}} \quad ----(*)$ $= \dfrac{1 + \sqrt{15}\mathrm{i}}{2+2\mathrm{i}} \quad or \quad \dfrac{1 - \sqrt{15}\mathrm{i}}{2+2\mathrm{i}}$ $= \dfrac{(1 + \sqrt{15}\mathrm{i})(2-2\mathrm{i})}{(2+2\mathrm{i})(2-2\mathrm{i})} \quad or \quad \dfrac{(1 - \sqrt{15}\mathrm{i})(2-2\mathrm{i})}{(2+2\mathrm{i})(2-2\mathrm{i})}$ $= \dfrac{(1+\sqrt{15}) + \mathrm{i}(-1+\sqrt{15})}{4} \quad or \quad \dfrac{(1-\sqrt{15}) + \mathrm{i}(-1-\sqrt{15})}{4}$	
	(ii) $z^4 - 2z^3 + z^2 + az + b = 0$ ---- (*) Sub $z = 1+2\mathrm{i}$ into (*), $(1+2\mathrm{i})^4 - 2(1+2\mathrm{i})^3 + (1+2\mathrm{i})^2 + a(1+2\mathrm{i}) + b = 0$ $(-7-24\mathrm{i}) - 2(-11-2\mathrm{i}) + (-3+4\mathrm{i}) + a(1+2\mathrm{i}) + b = 0$ $(12+a+b) + (2a-16)\mathrm{i} = 0$ Comparing the real and imaginary coefficients: $2a - 16 = 0$ ---- (1) $12 + a + b = 0$ ---- (2) Solving (1) & (2): $a = 8,\ b = -20$ Therefore $z^4 - 2z^3 + z^2 + 8z - 20 = 0$ Using GC: $z = 1+2\mathrm{i},\ 1-2\mathrm{i},\ 2,\ -2$	
	(ii) Alternative Method $\mathrm{f}(z) = z^4 - 2z^3 + z^2 + 8z - 20$ Since 1 + 2i is a root of f(z)=0, and the polynomial have all	

	real coefficients, this imply 1 − 2i is also a root. Hence $z^4-2z^3+z^2+az+b=(z-(1+2i))(z-(1-2i))(z^2+pz+q)$ $=((z-1)-2i)((z-1)+2i)(z^2+pz+q)$ $=((z-1)^2+4)(z^2+pz+q)$ $=(z^2-2z+5)(z^2+pz+q)$ Equating coeff of z^3 in f(z): $-2 = p-2$. Hence $p = 0$ Equating coeff of z^2 in f(z): $1 = q - 2p + 5$. Hence $q = -4$ Now $z^4-2z^3+z^2+az+b=(z^2-2z+5)(z^2-4)$ Equating coeff of z in f(z): $a = 8$ Equating constant in f(z): $b = -20$ **Hence** $a = 8$ and $b = -20$ $f(z)=0$ $(z^2-2z+5)(z^2-4)=0$ Hence the other 2 roots are ± 2	
	$z^4-2z^3+z^2+az+b=0$ ----(1) Let $z=-iw$ $w^4-2iw^3-w^2-8iw-20=0$ ----(2) Therefore $w=\frac{1}{-i}z \Rightarrow w=iz$ $w=i(1+2i),\ i(1-2i),\ i(2),\ i(-2)$ $w=i-2,\ i+2,\ 2i,\ -2i$	

Question 7 [10] Integration		
(i)	When the left height of a rectangle is used, area of region $=\frac{\pi}{10}\left(0+\sin^6\frac{\pi}{10}+\sin^6\frac{2\pi}{10}+\sin^6\frac{3\pi}{10}+\sin^6\frac{4\pi}{10}\right)$ $=\frac{\pi}{10}(1.0625)=0.10625\pi$ When the right height of a rectangle is used, area of region $=\frac{\pi}{10}\left(\sin^6\frac{\pi}{10}+\sin^6\frac{2\pi}{10}+\sin^6\frac{3\pi}{10}+\sin^6\frac{4\pi}{10}+\sin^6\frac{5\pi}{10}\right)$ $=\frac{\pi}{10}(2.0625)=0.20625\pi$ Thus, $0.10625\pi < A < 0.20625\pi$.	
(ii)	$I_n=\int_0^{\frac{\pi}{2}}\sin^{2n}x\,\mathrm{d}x$ $=\int_0^{\frac{\pi}{2}}\sin x\sin^{2n-1}x\,\mathrm{d}x$ $=\left[-\cos x\sin^{2n-1}x\right]_0^{\frac{\pi}{2}}+\int_0^{\frac{\pi}{2}}\cos x(2n-1)\sin^{2n-2}x\cos x\,\mathrm{d}x$ $=(2n-1)\int_0^{\frac{\pi}{2}}\cos^2 x\sin^{2n-2}x\ \mathrm{d}x$ $=(2n-1)\int_0^{\frac{\pi}{2}}(1-\sin^2 x)\sin^{2n-2}x\ \mathrm{d}x$ $=(2n-1)\int_0^{\frac{\pi}{2}}\sin^{2n-2}x-\sin^{2n}x\,\mathrm{d}x$ $=(2n-1)(I_{n-1}-I_n)$ $\Rightarrow I_n+(2n-1)I_n=(2n-1)I_{n-1}$ $\Rightarrow I_n=\frac{2n-1}{2n}I_{n-1}$	

	I_2 $= \frac{3}{4} I_1$ $= \left(\frac{3}{4}\right)\left(\frac{1}{2}\right) I_0$ $= \left(\frac{3}{4}\right)\left(\frac{1}{2}\right) \int_0^{\frac{\pi}{2}} 1 \, \mathrm{d}x,$ $= \left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{\pi}{2}\right)$ $= \frac{3}{16}\pi$ (Shown) Area of region I_3 $= \frac{5}{6} I_2$ $= \left(\frac{5}{6}\right)\left(\frac{3}{16}\pi\right)$ $= \frac{5}{32}\pi$	

8	Solution [13] Vectors	
	(i) $p_1:\ \mathbf{r}\bullet\begin{pmatrix}1\\5\\4\end{pmatrix}=4,\quad p_2:\mathbf{r}\bullet\begin{pmatrix}1\\-1\\1\end{pmatrix}=4$ Since the two normal vectors are not parallel to each other, the 2 planes are not parallel and hence intersecting. From GC, $x=4-\frac{3}{2}\lambda\quad -----(1)$ $y=-\frac{1}{2}\lambda\quad -----(2)$ $z=\ \lambda\quad -----(3)$ Hence $\ell:\ \mathbf{r}=\begin{pmatrix}4\\0\\0\end{pmatrix}+\lambda\begin{pmatrix}3\\1\\-2\end{pmatrix}$ where $\lambda\in\mathbb{R}$ $\frac{x-4}{3}=y=-\frac{z}{2}$	
	(ii) p_3 contains $\ell:\ \mathbf{r}=\begin{pmatrix}4\\0\\0\end{pmatrix}+\lambda\begin{pmatrix}3\\1\\-2\end{pmatrix}$ and point $Q(5,3,-6)$. Let T denote the point $(4,0,0)$. $\overrightarrow{QT}=\begin{pmatrix}4\\0\\0\end{pmatrix}-\begin{pmatrix}5\\3\\-6\end{pmatrix}=\begin{pmatrix}-1\\-3\\6\end{pmatrix}$ and $\begin{pmatrix}3\\1\\-2\end{pmatrix}$ are 2 direction vectors parallel to p_3. Consider $\begin{pmatrix}-1\\-3\\6\end{pmatrix}\times\begin{pmatrix}3\\1\\-2\end{pmatrix}=\begin{pmatrix}0\\16\\8\end{pmatrix}=8\begin{pmatrix}0\\2\\1\end{pmatrix}$ Therefore $\mathbf{n}=\begin{pmatrix}0\\2\\1\end{pmatrix}$ is a normal vector to p_3.	

	$\mathbf{r}\cdot\mathbf{n}=\begin{pmatrix}4\\0\\0\end{pmatrix}\cdot\begin{pmatrix}0\\2\\1\end{pmatrix}=0$ $p_3\ :\ \mathbf{r}\cdot\begin{pmatrix}0\\2\\1\end{pmatrix}=0$ $p_3\ :\ 2y+z=0$ *Geometrical Relationship:* <u>3 planes</u>/intersect at the common line l.	
	(iii) $p_2:\mathbf{r}.\begin{pmatrix}1\\-1\\1\end{pmatrix}=4$ ---- (1) Let N be the foot of perpendicular of $S(0,2,0)$ on p_2. Let l_{NS} be the line passing through points N and S. $l_{NS}:\mathbf{r}=\begin{pmatrix}0\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\1\end{pmatrix},\ \lambda\in\mathbb{R}$ ---- (2) Sub (2) into (1): $\left[\begin{pmatrix}0\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}\right].\begin{pmatrix}1\\-1\\1\end{pmatrix}=4$ ---- (*) $-2+3\lambda=4$ $\lambda=2$ $\overrightarrow{ON}=\begin{pmatrix}0\\2\\0\end{pmatrix}+2\begin{pmatrix}1\\-1\\1\end{pmatrix}=\begin{pmatrix}2\\0\\2\end{pmatrix}$ ---- (2) Let S' be the point of reflection of S in p_2. N is the midpoint of SS'. $\overrightarrow{ON}=\frac{1}{2}\left(\overrightarrow{OS}+\overrightarrow{OS'}\right)$ $\overrightarrow{OS'}=2\overrightarrow{ON}-\overrightarrow{OS}$	

	$\overrightarrow{OS'} = 2\begin{pmatrix}2\\0\\2\end{pmatrix} - \begin{pmatrix}0\\2\\0\end{pmatrix} = \begin{pmatrix}4\\-2\\4\end{pmatrix}$ Let m' be the line of reflection of m in p_2. The point $T(4,0,0)$ lying on m also lies on p_2. Therefore $T(4,0,0)$ also lies on m'. m' passes through $T(4,0,0)$ and $S'(4,-2,4)$ $m' : \mathbf{r} = \begin{pmatrix}4\\0\\0\end{pmatrix} + \lambda\left[\begin{pmatrix}4\\0\\0\end{pmatrix} - \begin{pmatrix}4\\-2\\4\end{pmatrix}\right],\ \lambda \in \mathbb{R}$ $m' : \mathbf{r} = \begin{pmatrix}4\\0\\0\end{pmatrix} + \lambda\begin{pmatrix}0\\2\\-4\end{pmatrix},\ \lambda \in \mathbb{R}$ $m' : \mathbf{r} = \begin{pmatrix}4\\0\\0\end{pmatrix} + \lambda\begin{pmatrix}0\\1\\-2\end{pmatrix},\ \lambda \in \mathbb{R}$	

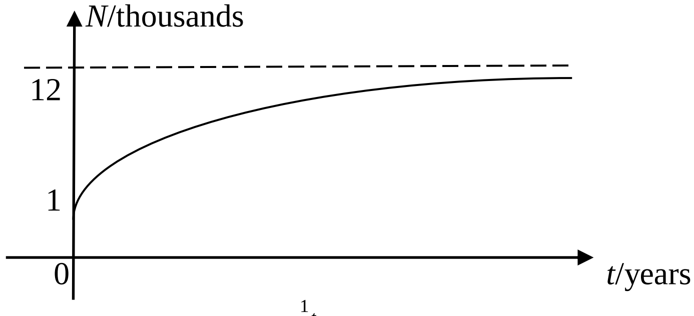
<table>
<tr><td>9</td><td>Solution [13] APGP</td><td></td></tr>
<tr><td></td><td>(i)
We first observe the following pattern:
<table><tr><td>Month</td><td>Beginning ($)</td><td>End ($)</td></tr><tr><td>Jan’ 2019</td><td>5000</td><td>5000×1.01</td></tr><tr><td>Feb’2019</td><td>$5000\times1.01-100$</td><td>$(5000\times1.01-100)\times1.01$</td></tr><tr><td>Mar’2019</td><td>$5000\times1.01^2-100$
$\times1.01-100$</td><td>$(5000\times1.01^2-100\times1.01$
$-100)\times1.01$</td></tr></table>
Thus, by the end of March 2019, John’s account is left with
$\$(5000\times1.01^3-100\times1.01^2-100\times1.01)$
$= \$4948.50$</td><td></td></tr>
<tr><td></td><td>(ii)
From (i), we can deduce that by the end of the n^{th} month, the money left in John’s saving account is
$5000\times1.01^n-\left(100\times1.01^{n-1}+100\times1.01^{n-2}+....+100\times1.01\right)$
$=5000\times1.01^n-100\times1.01\left(1+1.01+1.01^2+...+1.01^{n-2}\right)$
$=5000\times1.01^n-100\times1.01\times\frac{1\left(1.01^{n-1}-1\right)}{1.01-1}$
$=5000\times1.01^n-10000\times1.01\times\left(1.01^{n-1}-1\right)$
$=10100+5000\times1.01^n-10000\times1.01^n$
$=100\left(101-50\times1.01^n\right)$ (shown)</td><td></td></tr>
<tr><td></td><td>(iii)
Consider $100\left(101-50\times1.01^n\right)\le0$
$\Rightarrow 50\times1.01^n\ge101$
$\Rightarrow 1.01^n\ge\frac{101}{50}$
$\Rightarrow n\ge70.66$
Thus, it will take Mr Tan 71 months to deplete his saving account and it will be by November of 2024.

Alternative solution using table:
<table><tr><td>n</td><td>Amt left in account</td></tr><tr><td>70</td><td>$66.18>0$</td></tr><tr><td>71</td><td>$-34.16<0$</td></tr></table>Account depleted in the 71st month.</td><td></td></tr>
</table>

	(iv) The amount of interest earned by Mrs Tan's for each subsequent month forms an AP: 10, 10+5, 10+2×5,….. i.e. an AP with first term 10 and common difference 5 Thus, by the end of n^{th} month, the total amount of money in Mrs Tan's saving account $=3000+50n+\frac{n}{2}\left[2(10)+5(n-1)\right]$ $= =3000+50n+10n+2.5n(n-1)$ $=3000+57.5n+2.5n^2$ For Mrs Tan's account to be more than Mr Tan's, we let $3000+57.5n+2.5n^2 > 100\left(101-50\times1.01^n\right)$ ---- (*) Then using GC: we have (see table below) It takes 15 months from Jan 2019 for Mrs' Tan's account to exceed that of Mr Tan. Thus, it is by end of March 2020 that Mrs Tan's account will first be more than that of Mr Tan's.	

n	LHS of (*)	RHS of (*)
14	4295	4352.6
15	4425	4295.2
16	4560	4327.1

10	Solution [13] DE	
	(i) Based on the given information, we have $\frac{\mathrm{d}N}{\mathrm{d}t} = 2 - kN$, $k \in R$ Since it is given that $\frac{\mathrm{d}N}{\mathrm{d}t} = 1.5$ when $N = 3$, we have 1.5 = 2 –3k $\Rightarrow$ $k = \frac{1}{6}$. Thus, $\frac{\mathrm{d}N}{\mathrm{d}t} = 2 - \frac{1}{6}N \Rightarrow 6\frac{\mathrm{d}N}{\mathrm{d}t} = 12 - N$ (shown)	
	(ii) Now, $6\frac{\mathrm{d}N}{\mathrm{d}t} = 12 - N$ $\Rightarrow \int \frac{\mathrm{d}N}{12 - N} = \int \frac{1}{6}\ \mathrm{d}t$ $\Rightarrow \frac{\ln\|12 - N\|}{-1} = \frac{1}{6}t + c$ $\Rightarrow \ln\|12 - N\| = -\frac{1}{6}t + C$ Then, we have $\|12 - N\| = e^{-\frac{1}{6}t + C}$ $\Rightarrow 12 - N = \pm\ e^{-\frac{1}{6}t + C}$ $\Rightarrow 12 - N = Ae^{-\frac{1}{6}t}$ where $A = \pm e^{C}$ Next, given that when $t = 0$, $N = 1$, we have $A = 12 - 1 = 11$. Hence, the required equation connecting N and t is $N = 12 - 11e^{-\frac{1}{6}t}$	
	(iii) Let $12 - 11e^{-\frac{1}{6}t} \geq 6$	

	$11e^{-\frac{1}{6}t} \le 6$ $\Rightarrow e^{-\frac{1}{6}t} \le \frac{6}{11}$ $\Rightarrow -\frac{1}{6}t \le \ln\left(\frac{6}{11}\right)$ $\Rightarrow t \ge 3.64$ Thus, a minimum of 4 years are needed for the number of flying fox to first exceed 6000.	
	(iv)  We note that as $t \to \infty$, $e^{-\frac{1}{6}t} \to 0$. Thus, $N = 12 - 11e^{-\frac{1}{6}t} \to 12$ So, in the long run, the number of flying fox approaches 12 thousands.	
	(v) One possible limitation may be that the model fails to take into account external factors that affect the population. For example (i) outburst of sudden natural disasters which will affect population of the flying fox; (ii) the increase in the hunting of flying fox over the years; (iii) adverse climate change which affect their living habitat	