EUNOIA JUNIOR COLLEGE

JC2 Preliminary Examination 2019

General Certificate of Education Advanced Level

Higher 2

CANDIDATE NAME			
CLASS		INDEX NO.	

MATHEMATICS 9758/01

04 September 2019

Paper 1 [100 marks]

3 hours

Candidates answer on the Question Paper

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name, civics group and question number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.
Answer **all** questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **27** printed pages (including this cover page) and **1** blank page.

For markers' use:

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total

1 Electricity cost per household is calculated by multiplying the electricity consumption (in kWh), by the tariff (in cents/kWh). The tariff is set by the government and reviewed every 4 months.

The amount of electricity used by each household for each 4-month period, together with the total electricity cost for each household in the year, are given in the following table.

	Jan – April (in kWh)	May – Aug (in kWh)	Sept – Dec (in kWh)	Total electricity cost in the year ($)
Household 1	677	586	699	529.53
Household 2	1011	871	1048	790.63
Household 3	1349	1174	1417	1063.28

Write down and solve equations to find the tariff, in cents/kWh, to 2 decimal places, for each 4-month period. [4]

2 A string of fixed length l is cut into two pieces. The first piece is used to form a square of side s and the second piece is used to form a circle of radius r. Find the ratio of the length of the first piece to the second piece that gives the smallest possible combined area of the square and circle. [6]

3 A geometric progression has first term a and common ratio r, and an arithmetic progression has first term a and common difference d, where a and d are non-zero. The sums of the first 2 and 4 terms of the arithmetic progression are equal to the respective sums of the first 2 and 4 terms of the geometric progression.

(i) By showing $r^3 + r^2 - 5r + 3 = 0$, or otherwise, find the value of the common ratio. [5]

(ii) Given that $a < 0$ and the nth term of the geometric progression is positive, find the smallest possible value of n such that the nth term of the geometric progression is more than 1000 times the nth term of the arithmetic progression. [3]

4 **(i)** Find the series expansion for $(1+ax)^n$ in ascending powers of x, up to and including the term in x^3, where a is non-zero and $|a| < 1$. [1]

(ii) It is given that the coefficients of the terms in $x, x^2,$ and x^3 are three consecutive terms in a geometric progression. Show that $n = -1$. [2]

(iii) Show that the coefficients of the terms in the series expansion of $(1+ax)^{-1}$ form a geometric progression. [3]

(iv) Evaluate the sum to infinity of the coefficients of the terms in x of odd powers. [2]

5 **(a)** It is given that the equation $\mathrm{f}(x)=a$ has three roots x_1, x_2, x_3 where $x_1 < 0 < x_2 < x_3$, and a is a constant.

(i) How many roots does the equation $\mathrm{f}(|x|)=a$ have? With the aid of a diagram, or otherwise, explain your answer briefly. [2]

(ii) How many roots does the equation $\mathrm{f}(x-a)=a$ have? With the aid of a diagram, or otherwise, explain your answer briefly. [2]

(b) Solve the inequality $\frac{2\ln 2}{3\pi}x \le \left|\ln(1-\sin x)\right|$, where $0 \le x < 2\pi$. [4]

6 The curve C has equation $y=\frac{x^2+5x+3}{x+1}$.

(i) Show algebraically that the curve C has no stationary points. [2]

(ii) Sketch the curve C, indicating the equations of any asymptotes, and the coordinates of points where C intersects the axes. [4]

(iii) Region S is bounded by C, the y-axis, and the line $y=\frac{9}{2}$. Find the volume of the solid formed when region S is rotated about the x-axis completely. [3]

7 In the Argand diagram, the points P_1 and P_2 represent the complex numbers z and z^2 respectively, where $z=\sqrt{3}+\mathrm{i}\sqrt{3}$.

(i) Find the exact modulus and argument of z. [2]

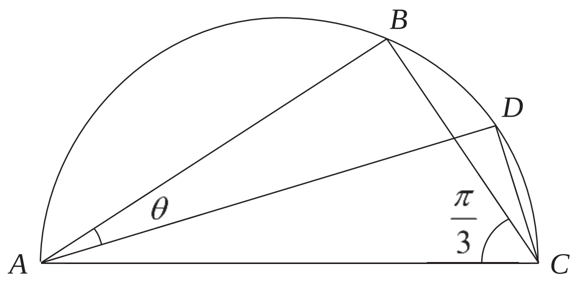
(ii) Mark the points P_1 and P_2 on an Argand diagram and find the area of the triangle OP_1P_2, where O represents the complex number 0. [3]

Let $w=2\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{3}\right)}$.

(iii) Find the set of integer values n such that $\arg\left(w^n z^3\right)=-\frac{\pi}{4}$. [4]

8 **(a)** Given that $2^y = 2 + \sin 2x$, use repeated differentiation to find the Maclaurin series for y, up to and including the term in x^2. [5]

(b)



The points A, B, C, and D lie on a semi-circle with AC as its diameter. Furthermore, angle $DAB = \theta$, and angle $ACB = \dfrac{\pi}{3}$.

(i) Show that $\dfrac{BC}{DC} = \dfrac{1}{\cos\theta - \sqrt{3}\sin\theta}$. [3]

(ii) Given that θ is a sufficiently small angle, show that

$$\frac{BC}{DC} \approx 1 + a\theta + b\theta^2,$$

for constants a and b to be determined. [3]

9 **(a)** **(i)** Find $\int 2\sin x \cos 3x \, dx$. [3]

(ii) Hence, show that $\int 2x \sin x \cos 3x \, dx = \dfrac{1}{16}\left[-4x\cos 4x + 8x\cos 2x + \sin 4x - 4\sin 2x\right] + C$, where C is an arbitrary constant. [3]

(b) The curve C has parametric equations

$$x = \theta^2, \; y = \sin\theta\cos 3\theta, \text{ where } 0 \le \theta \le \frac{\pi}{2}.$$

(i) Sketch the curve C, giving the exact coordinates of the points where it intersects the x-axis. [2]

(ii) By using the result in **(a)(ii)**, find the exact total area of the regions bounded by the curve C and the x-axis. [4]

10 An object is heated up by placing it on a hotplate kept at a high temperature. A simple model for the temperature of the object over time is given by the differential equation

$$\frac{dT}{dt} = k(T_H - T),$$

where T is the temperature of the object in degrees Celsius, T_H is the temperature of the hotplate in degrees Celsius, t is time measured in seconds and k is a real constant.

(i) State the sign of k and explain your answer. [1]

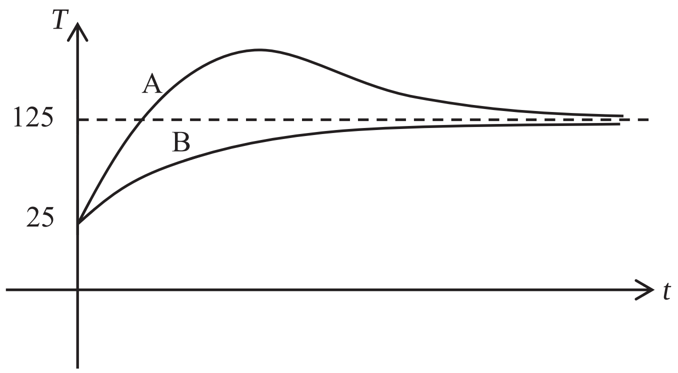
(ii) It is given that the temperature of the object is 25 degrees Celsius at $t = 0$, and the temperature of the hotplate is kept constant at 275 degrees Celsius. If the temperature of the object is 75 degrees Celsius at $t = 100$, find T in terms of t, giving the value of k to 5 significant figures. [6]

The model is now modified to account for heat lost by the object to its surroundings. The new model is given by the equation

$$\frac{dT}{dt} = k(T_H - T) - m(T - T_S),$$

where T_S is the temperature of the surrounding environment in degrees Celsius and m is a positive real constant.

(iii) It is given that the object eventually approaches an equilibrium temperature of 125 degrees Celsius, and that the surrounding environment has a constant temperature which is lower than 125 degrees Celsius. One of the two curves (A and B) shown below is a possible graph of the object's temperature over time. State which curve this is, and explain clearly why the other curve cannot be a graph of the object's temperature over time. [2]



(iv) Using the same value of k as found in part **(ii)** and assuming $T_S = 25$, find the value of m. (You need not solve the revised differential equation.) [3]

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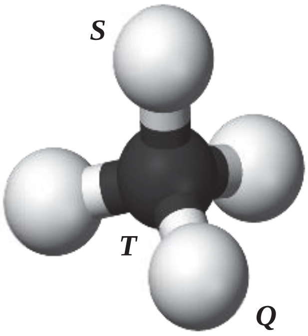

Methane (CH_4) is an example of a chemical compound with a tetrahedral structure. The 4 hydrogen (H) atoms form a regular tetrahedron, and the carbon (C) atom is in the centre.

Let the 4 H-atoms be at points *P*, *Q*, *R*, and *S* with coordinates $(9,2,9)$, $(9,8,3)$, $(3,2,3)$, and $(3,8,9)$ respectively.

(i) Find a Cartesian equation of the plane Π_1 which contains the points *P*, *Q* and *R*. [4]

(ii) Find a Cartesian equation of the plane Π_2 which passes through the midpoint of *PQ* and is perpendicular to $\overrightarrow{PQ}$. [2]

(iii) Find the coordinates of point *F*, the foot of the perpendicular from *S* to Π_1. [4]

(iv) Let *T* be the point representing the carbon (C) atom. Given that point *T* is equidistant from the points *P*, *Q*, *R* and *S*, find the coordinates of *T*. [3]

End of Paper